OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-6 : INVERSE TRIGONOMETRIC FUNCTION

UNIT TEST-1

1. If $y(x) = (x^x)^x$, $x > 0$, then $\frac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to ___________.

2. For the curve $C : (x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$, the value of $3y' - y^3y''$, at the point (α, α), $\alpha > 0$, on C is equal to _________.

Hints and Solutions

1. $\because y(x) = (x^x)^x$

$\therefore\ y = x^{x^2}$

$\therefore\ \frac{dy}{dx} = x^2.x^{x^2-1} + x^{x^2}\ \text{in}\, x.2x$

$\therefore\ \frac{dy}{dx} = \frac{1}{x^{x^2+1}(1+2\sin x)}$ ...(i)

Now, $\frac{d^2x}{dx} = \frac{d}{dx}((x^{x^2+1}(1+2\,\text{in}\,x))^{-1}).\frac{dx}{dy}$

$= \frac{-x(x^{x^2+1}(1+2\,\text{in}\,x))^{-2}.x^{x^2}(1+2\,\text{in}\,x)(x^2+2x^2\,\text{in}\,x+3)}{x^{x^2}(1+2\,\text{in}\,x)}$

$= \frac{-x^{x^2}(1+2\,\text{in}\,x)(x^2+3+2x^2\,\text{in}\,x)}{(x^{x^2}(1+2\,\text{in}\,x))^3}$

$\frac{d^2x}{dy^2(\text{at}\,x=1)} = -4$

$\frac{d^2x}{dy^2(\text{at}\,x=1)} + 20 = 16$

2. $\because C : (x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$ for point (α, α).

$\alpha^2 + \alpha^2 - 3 + (\alpha^2 - \alpha^2 - 1)^5 = 0$

$\therefore\quad \alpha = \sqrt{2}.$

On differentiating $(x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$ we get

$x + yy' + 5(x^2 - y^2 - 1)^4(x - yy') = 0$ ...(i)

When $x = y = \sqrt{2}$ then $y' = \frac{3}{2}$.

Again on differentiating eq. (i) we get :

$1 + (y')^2 + yy'' + 20(x^2 - y^2 - 1)(2x - 2yy')(x - y'y)$
$+ 5(x^2 - y^2 - 1)^4(1 - y'^2 - yy'') = 0$

For $x = y = \sqrt{2}.$ and $y' = \frac{3}{2}$.

we get $y'' = -\frac{23}{4\sqrt{2}}$

$\therefore\ 3y' - y^3y'' = 3.\frac{3}{2} - (\sqrt{2})^3.(-\frac{23}{4\sqrt{2}})$

$= 16$